

Categorical Studies of Token Structures: A Historical Perspective

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October 2026

Abstract

A *token structure* is a set of carrier elements together with a chosen collection of finite records over those elements and a fixed symbol alphabet, where structure-preserving maps are those functions that carry recorded tokens to recorded tokens. This survey traces the categorical treatment of such structures from four historical sources: categories of relational structures and their homomorphisms; topological and concrete categories, where morphisms are underlying functions subject to a fibrewise admissibility condition; algebraic and coalgebraic accounts of terms, term graphs, sharing and sequences; and the categorical linguistics descending from Lambek's syntactic calculus. We then describe how the Token Space framework of Pan assembles these ingredients into a single category whose objects are carriers with heaps of finite words over carrier and core symbols, and whose fibres are free quantales rather than products of powersets. For each component we state what the earlier theory already provides, what the Token Space presentation adds, and which structural questions remain open. The aim is a map of priorities and dependencies, not an evaluation of applications to machine learning.

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1 Introduction

Category theory was shaped by the structures it first observed: groups, modules, topological spaces, sheaves. The morphisms of those categories are functions or continuous maps whose admissibility is decided by a fixed, globally specified axiom set. By contrast, the data handled in symbolic computation and in modern language models consists of *tokens*: finite sequences of symbols drawn from a carrier and from a fixed alphabet, carrying structure that is often selected after the fact rather than declared in advance.

It is natural to ask whether there is a categorical theory proper to such token structures, and if so, which parts of it are inherited from earlier mathematics. This survey addresses that question historically. Our working definition is the following.

Definition 1.1 (Token structure). *Fix a set E of core symbols. A token structure over E is a pair (B, Υ) where B is a set (the carrier) and $\Upsilon \subseteq (B \sqcup E)^*$ is a set of finite words (the heap). A token map $(B, \Upsilon) \rightarrow (B', \Upsilon')$ is a function $f : B \rightarrow B'$ such that the induced map $f^* : (B \sqcup E)^* \rightarrow (B' \sqcup E)^*$, acting as f on carrier letters and as the identity on E , satisfies $f^*(\Upsilon) \subseteq \Upsilon'$.*

This is the elementary category $\mathbf{TS}_E$ of Pan’s Token Space [28], stated in the notation of the present survey. It will serve as the fixed point of comparison. Four lines of earlier work meet in it:

1. **Relational structures.** Reading each word shape as a relation symbol, $\mathbf{TS}_E$ is a category of relational structures over a (possibly infinite) finitary signature. The categorical study of such categories is classical and includes the exponentiability and quasitopos results that $\mathbf{TS}_E$ inherits (section 2).
2. **Topological and concrete categories.** The forgetful functor $\mathbf{TS}_E \rightarrow \mathbf{Set}$ is topological: every family of carrier functions has initial and final lifts. Initial, final, preserving, reflecting, and heap-final maps are instances of concepts developed for **Top**, **Rel** and measurable spaces (section 3).
3. **Terms, term graphs and coalgebra.** The treatment of trees, substitution, sharing and sequential behaviour draws on algebraic theories, gs-monoidal categories, term graph rewriting and universal coalgebra (section 4).
4. **Categorical linguistics.** The intent to carry linguistic meaning in categorical structure dates from Lambek’s syntactic calculus and continues in pregroup grammars and compositional distributional semantics (section 5).

Section 6 describes how Token Space combines these, and section 7 isolates the single feature that none of the four sources covers directly: the heap lives in a free monoid, so each fibre is a free quantale with concatenation, and core symbols occur inside records rather than as relation

names. Section 8 lists structural questions whose answers would decide whether this feature supports theorems of its own.

A caution on method. Throughout we distinguish three claims that are easily conflated: that a structure can be *encoded* in a given framework, that the framework *organizes* known results, and that the framework yields *new theorems*. Encoding is cheap, since every function is a relation and every relation is a set of tokens; organization has value but is not priority; theorems require a comparison category and a universal property. We attribute each result to the earliest source we have verified, and we mark explicitly where a chain of priority has not been fully traced.

2 Relational structures and their categories

2.1 Homomorphisms of relational structures

The modern notion of a relational structure and homomorphism is due to model theory; Hodges [17] and Adámek–Rosický [3] give standard accounts. For a signature Σ of relation symbols with arities, a Σ -structure is a set A with relations $R^A \subseteq A^{k_R}$, and a homomorphism preserves each relation: $(a_1, \dots, a_k) \in R^A$ implies $(fa_1, \dots, fa_k) \in R^B$. Homomorphisms preserve but need not reflect; a *strong* homomorphism reflects as well. The category $\mathbf{Str}(\Sigma)$ is complete, cocomplete and locally finitely presentable.

Definition 1.1 is a presentation of $\mathbf{Str}(\Sigma_E)$ for the signature Σ_E whose symbols are the *shapes*: word length, positions occupied by carrier letters, and the fixed assignment of core symbols to the remaining positions. A word with no carrier letters is a nullary relation, a fact that is true or false of the structure as a whole. This translation is stated in [28, §5]; it is the key to attributing the basic categorical properties of $\mathbf{TS}_E$.

2.2 Exponentials and the quasitopos property

Categories of relational structures are not toposes but are quasitoposes. The general result is that for any relational signature, $\mathbf{Str}(\Sigma)$ is a quasitopos; exponentials are computed by equipping the set of homomorphisms with the relations that make evaluation a homomorphism. A detailed treatment appears in Parker [29], who also studies exponentiability inside full subcategories cut out by Horn-type constraints and shows that these need not inherit exponentials.

Pan [28, Thm. 5.2] proves that $\mathbf{TS}_E$ is bicartesian closed and not a topos, and later locates the result as an instance of the quasitopos theorem via the shape translation. The explicit formulas for exponentials in $\mathbf{TS}_E$ are useful, but the classification is inherited. The same remark applies to the observation that full subcategories of “algebraic” token structures (graphs of operations recorded as tokens) need not be closed under the ambient exponential [28, §6]: this is Parker’s phenomenon for constrained subcategories.

2.3 Relational structures as a universal encoding

That every finitary algebraic structure embeds fully and faithfully into relational structures by recording operation graphs is standard; it is the usual passage from functional to relational signatures in model theory [17]. Pan’s “algebraic tokenization” [28, Thm. 6.3] is this embedding stated for token structures, with an explicit account of which limits are preserved. The contribution is the uniform encoding into one record format; the full faithfulness is inherited.

2.4 Relations as morphisms: allegories and Rel

A different tradition takes relations as morphisms rather than as structure on objects. Freyd and Scedrov’s allegories [14] axiomatize categories of relations with converse and meet; **Rel**

is a dagger category in Selinger’s sense [33]. This tradition is relevant to token structures by contrast: a token map preserves but need not reflect, so $\mathbf{TS}_E$ has no converse operation, and the graph functor $\mathbf{TS}_E \rightarrow \mathbf{Rel}$ forgets precisely the directional information carried by the heap. The observation that preservation without reflection is the absence of a dagger is elementary; we are not aware of it being stated for relational structure categories before the Token Space supplements of 2026, but we have not searched exhaustively.

3 Topological and concrete categories

3.1 Topological functors

A concrete category $U : \mathbf{C} \rightarrow \mathbf{Set}$ is *topological* if every U -structured source has a unique initial lift; dually every structured sink has a final lift. The theory is set out by Adámek, Herrlich and Strecker [2], with roots in Wyler [40] and Herrlich [16]. **Top**, **Rel** (sets with a binary relation), preordered sets, measurable spaces, and every $\mathbf{Str}(\Sigma)$ are topological over **Set**.

Consequently $\mathbf{TS}_E \rightarrow \mathbf{Set}$ is topological. Its fibre over a carrier B is the complete lattice of heaps $\mathcal{P}((B \sqcup E)^*)$; the initial lift of a family $f_i : B \rightarrow B_i$ is the heap $\bigcap_i (f_i^*)^{-1}(\Upsilon_i)$ and the final lift of a family $g_j : B_j \rightarrow B$ is $\bigcup_j g_j^*(\Upsilon_j)$. Pan’s statements that the least target heap making given functions token maps is the union of transported heaps, and that the greatest source heap is the intersection of inverse images [28, §5], are the initial and final lift formulas.

3.2 Initial, final and strong morphisms

The following dictionary summarizes the correspondence between the Token Space vocabulary and the standard vocabulary of topological categories [2].

Token Space [28]	Topological categories [2]
token map (preserves heap)	morphism
preserves and reflects	initial morphism; strong homomorphism
heap-final ($\Upsilon' = f^*\Upsilon$)	final morphism
surjective and heap-final	regular epimorphism; quotient
injective and reflecting	regular monomorphism; embedding
least target heap	final lift
greatest source heap	initial lift

Two consequences deserve mention. First, since $\mathbf{TS}_E$ is a quasitopos and hence locally cartesian closed, its effective descent morphisms are exactly its regular epimorphisms, by the general theorem for locally cartesian closed categories (see Janelidze and Tholen [20]). Thus “descent” statements about heap-final surjections in token structures are instances of this theorem. Second, the quotient of a token structure by an equivalence relation on the carrier, with the transported heap, is the final lift of the quotient map; Pan uses this in the study of behavioural quotients [28, §13].

3.3 Chu spaces

Chu spaces [30, 5] offer an alternative carrier-with-structure framework in which objects are matrices $A \times X \rightarrow K$ and morphisms are adjoint pairs. Pratt established broad representation theorems for Chu spaces, including the full embedding of all topological and relational categories over **Set**. Any representation theorem for token structures must therefore be compared with Chu representation, and a 2026 comparison [37] concludes that token structures cannot be claimed to be more general than Chu spaces, since Chu’s global test functions express constraints that finite records do not.

4 Terms, term graphs, sharing and coalgebra

4.1 Algebraic theories and substitution

Lawvere’s algebraic theories [25] present syntax as a category with finite products in which substitution is composition. Free algebras and term substitution are the paradigm for “internal structure that changes under maps”. Token Space’s tree token structures and leaf substitution [28, §8] belong to this lineage; its contribution is to show that, under a *supported* condition and with root and empty-tuple markers, tree substitution corresponds bijectively to ordinary token maps between reified records (Pan’s reification theorem). The reification bridge is shared with the companion paper on causal token structures [11].

4.2 Term graphs and gs-monoidal categories

Sharing of subterms is the subject of term graph rewriting. Corradini and Gadducci [10] characterize term graphs as arrows of free gs-monoidal categories; Hasegawa [15] develops categorical semantics of sharing graphs including cyclic sharing. Bahr [4] studies categories of term graphs and their homomorphisms, observing that in the rooted case homomorphisms are unique when they exist, so the category is a preorder.

Pan’s executable graph fragment [28, §§9–10]—nodes with primitive labels and ordered operands, ordered input and output ports, maps preserving labels and operands—is in exact correspondence with ported term graphs over a ranked signature, as a 2026 audit established by a full and faithful translation [36]. The general category $\mathbf{TS}_E$ is strictly larger: it admits parallel non-identical endomorphisms and need not have unique node definitions. The distinction is between a framework that contains term graphs as a constrained fragment and one that coincides with them.

4.3 Universal coalgebra and behavioural equivalence

Rutten’s universal coalgebra [32] supplies the general theory of state-based systems, homomorphisms, bisimulation and final semantics. Jacobs’ monograph [19] develops *relation lifting*: for an endofunctor F on $\mathbf{Set}$, relations on B lift to relations on $F(B)$, and F -structures are pairs $(B, \Upsilon \subseteq F(B))$ with maps those functions whose F -image restricts. The construction of $\mathbf{TS}_E$ as structures for the functor $F_E(B) = (B \sqcup E)^*$, written $\mathbf{SX}_{F_E}(\mathbf{Set})$ in [28], is an instance of this general template.

Pan’s result that structural indiscernibility with respect to a future-observation heap is behavioural equivalence, and that exact incremental encodings are right congruences contained in it [28, §13], places the Myhill–Nerode and Mealy-minimization circle of ideas into token language. The characterization itself is classical [32]; the token presentation specifies which observations are recorded.

4.4 Event structures and concurrency

Winskel’s event structures [38] and the later theory of stable families [39] treat configurations, local injectivity and rigid maps. Pan’s ready-cube analysis of how a sharing map changes exposed independence [28, §10] has been compared with these notions [36]: for conflict-free finite acyclic graphs, node-injectivity of a graph map is closely related to local injectivity on configurations. The dimension formula for images of ready cubes is a concrete computation within that setting.

4.5 State migration

The problem of changing a program’s representation during execution while recovering its live state is the subject of on-stack replacement; D’Elia and Demetrescu [12] give a distilled account

with definitions of strict OSR, live-variable equivalence and composition of migrations. Pan’s theorem that a node-surjective graph map induces a natural migration of reachable frontiers [28, Thm. 10.25] has been translated into a strict OSR instance for straight-line programs [36]. The kernel-inclusion criterion for exact reconstruction [28, Thm. 10.24] is the factorization of set-maps through their images.

5 Categorical linguistics

The intent to carry linguistic meaning in categorical structure has its own history. Lambek’s syntactic calculus [23] treats grammatical types as objects of a residuated monoidal category and parsing as derivation; Lambek later introduced pregroup grammars [24]. Coecke, Sadrzadeh and Clark [9] combine pregroup grammar with vector-space semantics through a monoidal functor, yielding compositional distributional semantics (DisCoCat). Spivak’s ologs [34] and the categorical treatment of databases by Johnson, Rosebrugh and Wood [21] use categories to organize linguistic and relational knowledge.

Token Space differs from these in its target: not a semantics for sentences, but a structural interface language for computations on token sequences. Where DisCoCat fixes a grammar category and a semantics category and studies functors between them, Token Space fixes a record format and asks which functions preserve selected records. The two are compatible rather than competing; no systematic comparison has been made, and the question of whether a pregroup derivation can be presented as a token structure in a way that makes the DisCoCat functor a token map is, to our knowledge, open.

6 The Token Space synthesis

6.1 Chronology

The first version of Token Space appeared on arXiv in April 2024 [27], with the stated question of finding a mathematical framework in which the meaning of tokens—grouping, order, parameter types—can be carried through the inputs and outputs of functions. That version already contained the carrier-with-heap definition, the tokenization of structures, and tree structures with reification. The fourth arXiv version [28], corresponding to the author’s sixteenth internal revision, adds the executable graph fragment, frontiers and state migration, the cache and eviction analysis for autoregressive models, and the behavioural quotient results. Companion manuscripts on causal token structures [11] and on layered construction with regular heaps [13] develop designated roots, gluing, and effective reasoning over regular heaps.

6.2 What the synthesis consists of

Relative to the four sources, the synthesis of [28] consists of the following choices.

1. **One record format for all structure.** Relations of any arity, operation graphs, tree edges, graph nodes, execution steps and observations are all words in $(B \sqcup E)^*$. A single category houses them.
2. **Deferred signature.** The shapes present in a heap determine the effective signature; no signature is declared in advance. This is a methodological inversion of the model-theoretic convention and is what allows the same object to be read under several signatures.
3. **Morphisms as obligations.** A function is admitted as a morphism exactly when it preserves the declared records. “Meaning” is thereby made into a membership condition on hom-sets rather than an annotation on functions.

4. **Separation of correctness notions.** For a change of representation, output preservation, exact state migration, future-observation equivalence, and preservation of concurrent choices are shown to be distinct conditions, with explicit counterexamples separating them [28, §10].

Each of these is an organizational contribution. Items 1 and 2 are design decisions; item 3 is the definition of a concrete category; item 4 is a set of theorems within the framework whose individual components have the antecedents listed in section 4.

6.3 Independent audits

Between October 2024 and October 2026 a sequence of internal audits compared Token Space with the literature cited above [36, 37]. Their consistent finding is that the elementary category and its basic properties are inherited, that the graph fragment coincides with ported term graphs, and that the migration and concurrency results have direct analogues in OSR and event structures. The audits also confirm that the proofs are correct under their stated hypotheses and that finite enumerations agree with them. We report these findings as part of the historical record; they are the reason this survey frames Token Space as a synthesis rather than a new foundation.

7 What is proper to token structures

After the attributions above, one feature of definition 1.1 is not covered by any of the four sources as they stand.

Remark 7.1 (The fibre is a free quantale). *In a category of Σ -structures the fibre over a carrier B is $\prod_{R \in \Sigma} \mathcal{P}(B^{k_R})$, a product of powersets with no multiplication. In $\mathbf{TS}_E$ the fibre is $\mathcal{P}((B \sqcup E)^*)$, the free quantale on the free monoid $(B \sqcup E)^*$: it carries concatenation of heaps, Kleene closure, and left and right quotients. The map f^* on words is a monoid homomorphism, and its direct image is a quantale homomorphism between fibres.*

Two consequences follow immediately and have no analogue for fixed-signature relational structures.

- **Core symbols occur inside records.** A relation name, a parameter type, a version tag and a constant can occupy positions in the same word. The “signature” is part of the heap, not a parameter of the category.
- **Regular heaps.** Since heaps are languages, one may restrict to regular heaps. The companion manuscript [13] shows that regular heaps form an effective Boolean hyperdoctrine with decidable guards; this concept has no counterpart for $\mathbf{Str}(\Sigma)$. The relevant prior theory is that of automatic structures [22, 7] and synchronous rational relations, not term graphs or OSR.

We stress that definition 7.1 is an observation about the construction, not a theorem. Quantale-enriched and quantaloid-enriched categories have been studied extensively [31, 35]; relation lifting for arbitrary **Set**-endofunctors is standard [19]. What we have not located is a study of the topological category generated by the specific functor $F_E(B) = (B \sqcup E)^*$ that exploits the monoid structure of its fibres. Whether this gap contains theorems is the subject of the next section.

8 Open structural questions

The following questions are posed so that an affirmative answer would be a result about token structures that is not an instance of the theories surveyed above. Each is accompanied by the prior work against which an answer must be checked.

Question 8.1 (Concatenation as a tensor). *Does heap concatenation induce a monoidal or bicategorical structure on $\mathbf{TS}_E$ distinct from the cartesian product, under which token maps are lax or strong monoidal? Compare with Rosenthal’s quantale modules [31] and Stubbe’s quantaloid-enriched categories [35].*

Question 8.2 (Freeness of the regular hyperdoctrine). *Is the Boolean hyperdoctrine of regular heaps [13] the free one, in a suitable 2-category of hyperdoctrines, on the alphabet E ? Compare with Lawvere’s hyperdoctrines [26] and with the automatic-structure literature [7].*

Question 8.3 (Change of core as a fibration). *Varying E gives a functor $E \mapsto \mathbf{TS}_E$. Because core symbols occur inside records, base change along $E \rightarrow E'$ interacts with carrier substitution in a way that differs from change of signature in $\mathbf{Str}(\Sigma)$. Is the resulting indexed category, in the sense of Jacobs [18], a fibration with properties not shared by signature change?*

Question 8.4 (Directed spans and the missing dagger). *Characterize the subcategory of $\mathbf{TS}_E$ on which the graph functor to $\mathbf{Rel}$ admits a dagger. The strong homomorphisms are a candidate; is the characterization exactly those, and does the failure of a dagger elsewhere have a universal description? Compare with allegories [14].*

Question 8.5 (Representation cost of hom-sets). *For finite token structures C, D , what is the representation cost of $\text{Hom}(C, D)$ in a fixed compilation language, as a function of the heap of C ? Partial results exist for binary signatures by importing knowledge-compilation lower bounds [8, 1]; Berkholtz and Vinall-Smeeth [6] study representations of homomorphism sets directly.*

Answers to these questions would be the first results we know of that depend on the free-quantale fibre rather than on properties shared with all topological or relational categories.

9 Conclusion

The categorical treatment of token structures has a longer history than its recent applications suggest. Its objects are relational structures; its morphism concepts come from topological categories; its trees, graphs, sharing and behaviour come from algebraic theories, term graphs and coalgebra; its linguistic ambition comes from Lambek. Token Space [28] is the first framework we know of to assemble these into one record language with a deferred signature and to use that language to separate several correctness conditions for changes of representation. Its elementary properties are inherited, and its graph fragment coincides with ported term graphs.

What the synthesis leaves genuinely unexplored is the consequence of placing all records in a free monoid: fibres that are free quantales, core symbols inside records, and regular heaps with decidable guards. Whether that feature yields categorical structure of its own is, at the time of writing, an open question rather than an established result. The honest historical position is therefore that token structures are a new *instance* of old constructions, whose proper theory—if it exists—has yet to be written.

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